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Chiral perturbation theory for tensor mesons*

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Abstract: Interactions of a_2, K_2^*, f_2 and f_2' tensor-mesons with low-energy π, K, η, η' pseudo-scalar mesons are constrained by chiral symmetry. We derive a chiral Lagrangian of tensor mesons in which the tensor mesons are treated as heavy non-relativistic matter fields. Using $1/N_c$ counting, we derive relations among unknown couplings of the chiral Lagrangian. Chiral perturbation theory is applied to the tensor-meson mass matrix. At one-loop there are large corrections to the individual tensor meson masses, but the singlet-octet mixing angle remains almost unchanged. We argue that all heavy mesons of spin ≥ 1 share common feature of chiral dynamics.

Keywords: Chiral Lagrangians, QCD.

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An important application of chiral perturbation theory is to describe the interaction of matter fields (such as nucleons [1, 2] or hadrons containing a heavy quark [3, 4, 5, 6]) with low-momentum pseudo-Goldstone bosons – the pions, kaons and eta. In ref. [7] chiral perturbation theory was used to describe the interactions of the ρ, K^*, ϕ and ω vector mesons with low-momentum pseudo-Goldstone bosons. In this article, we will extend the formalism to study the lowest-lying tensor meson nonet, which are treated as static heavy particles. This nonet is expected to have quantum numbers $J^P = 2^+$, and contains the isotriplet $a_2(1320)$ and the $S = \pm 1$ isodoublets $K_2^*(1430)$. The two isosinglet states are not as well established, but experimental evidences suggest that they are probably $f_2(1275)$ and $f_2'(1525)$. The mass difference between these nine lowest-lying tensor mesons are small compared to the chiral symmetry breaking scale $4\pi f_\pi \approx 1$ GeV. Hence, chiral perturbation theory should be applicable as a systematic expansion procedure for a class of processes involving tensor mesons and soft Goldstone bosons. In the past, chiral perturbation theory has been used extensively to study processes which do not have a tensor meson in the final state. In such decays, the final state pions are not soft enough that the application of the chiral Lagrangian to such processes is not justified a priori. At the best, they serve as a phenomenological model. To exemplify utility of the developed chiral perturbation theory, we calculate leading nonanalytic chiral corrections to mass spectrum and singlet-octet mixing angle. We note that leading nonanalytic corrections has been studied previously in [8], but without utilizing static heavy particle limit which simplifies the analysis considerably.

The pseudo-Goldstone boson fields can be written as a 3×3 special unitary matrix

$$\Sigma = \exp \frac{2i\Pi}{f}, \tag{1}$$

where

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{2}$$

Under chiral $SU(3)_L \times SU(3)_R$, $\Sigma \to L\Sigma R^\dagger$, where $L \in SU(3)_L$ and $R \in SU(3)_R$. At leading order in chiral perturbation theory, f can be identified with the pion or kaon decay constant ($f_\pi \sim 132$ MeV, $f_K \sim 160$ MeV). It is convenient, when describing the interactions of the pseudo-Goldstone bosons with other fields to introduce

$$\xi = \exp \frac{i\Pi}{f} = \sqrt{\Sigma}. \tag{3}$$

Under chiral $SU(3)_L \times SU(3)_R$,

$$\xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{4}$$

where in general U is a complicated function of L, R and the meson fields Π. For transformations $V = L = R$ in the unbroken $SU(3)_V$ subgroup, $U = V$.

The tensor meson fields are introduced as a 3×3 octet matrix

$$\tilde{\mathcal{O}}_{\mu\nu} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^{(8)}}{\sqrt{6}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^{(8)}}{\sqrt{6}} & K_2^{*0} \\ K_2^{*-} & \overline{K}_2^{*0} & -\frac{2f_2^{(8)}}{\sqrt{6}} \end{pmatrix}_{\mu\nu} , \tag{5}$$

and as a singlet

$$\tilde{\mathcal{S}}_{\mu\nu} = f_2^{(0)}{}_{\mu\nu} \,. \tag{6}$$

(We have deliberately made our notations for the tensor mesons identical to that for the vector mesons in ref. [7] except for an additional tilde.) By definition these tensor mesons are symmetric and traceless Lorentz tensors,

$$\begin{aligned} \tilde{\mathcal{O}}_{\mu\nu} &= \tilde{\mathcal{O}}_{\nu\mu} \,, \qquad \tilde{\mathcal{O}}^{\mu}_{\mu} = 0, \\ \tilde{\mathcal{S}}_{\mu\nu} &= \tilde{\mathcal{S}}_{\nu\mu} \,, \qquad \tilde{\mathcal{S}}^{\mu}_{\mu} = 0 \,. \end{aligned} \tag{7}$$

Moreover, the polarizations of the tensor mesons are necessarily orthogonal to the momentum,

$$p^\mu \tilde{\mathcal{O}}_{\mu\nu} = p^\mu \tilde{\mathcal{S}}_{\mu\nu} = 0 \,. \tag{8}$$

Under chiral $SU(3)_L \times SU(3)_R$,

$$\tilde{\mathcal{O}}_{\mu\nu} \to U \tilde{\mathcal{O}}_{\mu\nu} U^\dagger \,, \qquad \tilde{\mathcal{S}}_{\mu\nu} \to \tilde{\mathcal{S}}_{\mu\nu} \,, \tag{9}$$

and under charge conjugation,

$$C \tilde{\mathcal{O}}_{\mu\nu} C^{-1} = \tilde{\mathcal{O}}^T_{\mu\nu} \,, \qquad C \tilde{\mathcal{S}}_{\mu\nu} C^{-1} = \tilde{\mathcal{S}}_{\mu\nu} \,, \qquad C \xi C^{-1} = \xi^T \,. \tag{10}$$

We construct a chiral lagrangian for tensor mesons by treating the tensor mesons as heavy static fields [9, 10] with fixed four-velocity v_μ, with $v^2 = 1$. Eq. (8) becomes

$$v^\mu \tilde{\mathcal{O}}_{\mu\nu} = v^\mu \tilde{\mathcal{S}}_{\mu\nu} = 0 \,. \tag{11}$$

The chiral lagrangian density which describes the interactions of the tensor mesons with the low-momentum π, K and η mesons has the general structure

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm mass} \,. \tag{12}$$

At the leading order in the derivative and quark mass expansions,

$$\mathcal{L}_{\rm kin} = -\frac{i}{2} \tilde{\mathcal{S}}^\dagger_{\mu\nu} (v \cdot \partial) \tilde{\mathcal{S}}^{\mu\nu} - \frac{i}{2} \,{\rm Tr}\, \tilde{\mathcal{O}}^\dagger_{\mu\nu} (v \cdot \mathcal{D}) \tilde{\mathcal{O}}^{\mu\nu} \,, \tag{13}$$

and

$$\begin{aligned} \mathcal{L}_{\rm int} = {} & \tfrac{i}{2} \tilde{g}_1 \tilde{\mathcal{S}}^\dagger_{\mu\rho} \,{\rm Tr}\, (\tilde{\mathcal{O}}^\rho_\nu A_\lambda) v_\sigma \epsilon^{\mu\nu\lambda\sigma} + h.c. \\ & + \tfrac{i}{2} \tilde{g}_2 \,{\rm Tr}\, (\{\tilde{\mathcal{O}}^\dagger_{\mu\rho}, \tilde{\mathcal{O}}^\rho_\nu\} A_\lambda) v_\sigma \epsilon^{\mu\nu\lambda\sigma} \,, \end{aligned} \tag{14}$$

where

$$\mathcal{D}_\lambda \tilde{\mathcal{O}}_{\mu\nu} = \partial_\lambda \tilde{\mathcal{O}}_{\mu\nu} + [V_\lambda, \tilde{\mathcal{O}}_{\mu\nu}] \tag{15}$$

and

$$V_\lambda = \tfrac{1}{2}(\xi\partial_\lambda\xi^\dagger + \xi^\dagger\partial_\lambda\xi)\,, \qquad A_\lambda = \tfrac{i}{2}(\xi\partial_\lambda\xi^\dagger - \xi^\dagger\partial_\lambda\xi)\,. \tag{16}$$

Finally, to linear order in quark mass expansion,

$$\begin{aligned}\mathcal{L}_{\text{mass}} = \ & \frac{\tilde{\mu}_0 + \tilde{\sigma}_0 \operatorname{Tr}\mathcal{M}}{2} \tilde{\mathcal{S}}^\dagger_{\mu\nu}\tilde{\mathcal{S}}^{\mu\nu} + \frac{\tilde{\mu}_8 + \tilde{\sigma}_8 \operatorname{Tr}\mathcal{M}}{2} \operatorname{Tr}\tilde{\mathcal{O}}^\dagger_{\mu\nu}\tilde{\mathcal{O}}^{\mu\nu} \\ & + \frac{\tilde{\lambda}_1}{2} \operatorname{Tr}(\tilde{\mathcal{O}}^\dagger_{\mu\nu}\mathcal{M}_\xi)\tilde{\mathcal{S}}^{\mu\nu} + h.c. + \frac{\tilde{\lambda}_2}{2}\operatorname{Tr}(\{\tilde{\mathcal{O}}^\dagger_{\mu\nu}, \tilde{\mathcal{O}}^{\mu\nu}\}\mathcal{M}_\xi)\,,\end{aligned} \tag{17}$$

where $\mathcal{M}$ is the quark mass matrix $\mathcal{M} = \operatorname{diag}(m_u, m_d, m_s)$, and

$$\mathcal{M}_\xi = \tfrac{1}{2}(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger)\,. \tag{18}$$

As mentioned in ref. [7], one of the mass parameters can be removed by a simultaneous phase redefinition of $\mathcal{O}$ and $\mathcal{S}$, and only the singlet-octet mass difference $\Delta\tilde{\mu} \equiv \tilde{\mu}_0 - \tilde{\mu}_8$ is physically relevant. It is yet unclear which f_2 state corresponds to the lowest-lying isosinglet tensor meson, but for all reasonable choices $\Delta\tilde{\mu} \leq 300$ MeV and can be regarded as a quantity of order m_q. Note that $\Delta\tilde{\mu}$ is of order N_c^{-1} and vanishes in the large N_c limit.

To fix couplings in the chiral Lagrangian, we analyze the spectrum of tensor mesons given at leading order in chiral perturbation theory. The analysis is essentially identical to the SU(3) analysis. In the approximation with exact isospin symmetry $m_u = m_d = \hat{m}$, we find that a_2 and K_2^* tensor mesons are interaction and energy eigenstates simultaneously and have masses as:

$$\begin{aligned} M_{a_2} &= \overline{\mu}_8 + 2\tilde{\lambda}_2\hat{m} \\ M_{K_2^*} &= \overline{\mu}_8 + \tilde{\lambda}_2(\hat{m} + m_s)\,, \end{aligned} \tag{19}$$

while the $f^{(0)}$ and $f^{(8)}$ mesons have mass matrix

$$M^{(8-0)} = \begin{pmatrix} \overline{\mu}_8 + \frac{2}{3}\tilde{\lambda}_2(\hat{m} + 2m_s) & \frac{2}{\sqrt{6}}\tilde{\lambda}_1(\hat{m} - m_s) \\ \frac{2}{\sqrt{6}}\tilde{\lambda}_1^*(\hat{m} - m_s) & \overline{\mu}_0 \end{pmatrix}. \tag{20}$$

We have abbreviated combinations of parameters as

$$\overline{\mu}_8 = \tilde{\mu}_8 + \tilde{\sigma}_8 \operatorname{Tr}\mathcal{M} \qquad \overline{\mu}_0 = \tilde{\mu}_0 + \tilde{\sigma}_0 \operatorname{Tr}\mathcal{M}\,. \tag{21}$$

Using the above relations, we find that the singlet-octet mass matrix elements can be expressed in terms of the experimentally measured tensor-meson masses:

$$\begin{aligned} M_{11}^{(8-0)} &= \frac{4}{3}M_{K_2^*} - \frac{1}{3}M_{a_2} \\ M_{22}^{(8-0)} &= M_{f_2'} + M_{f_2} - \frac{4}{3}M_{K_2^*} + \frac{1}{3}M_{a_2} \\ M_{12}^{(8-0)} = M_{21}^{(8-0)} &= \pm\Big[\Big(\frac{4}{3}M_{K_2^*} - \frac{1}{3}M_{a_2} - M_{f_2'}\Big)\cdot\Big(M_{f_2} - \frac{4}{3}M_{K_2^*} + \frac{1}{3}M_{a_2}\Big)\Big]\,. \end{aligned} \tag{22}$$

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The mass eigenstates of octet-singlet mixing tensor mesons are parametrized by a mixing angle Θ_T:

$$\begin{pmatrix} |f_2\rangle \\ |f_2'\rangle \end{pmatrix} = \begin{pmatrix} \cos\Theta_T & +\sin\Theta_T \\ -\sin\Theta_T & \cos\Theta_T \end{pmatrix} \begin{pmatrix} |f_2^{(8)}\rangle \\ |f_2^{(0)}\rangle \end{pmatrix}. \tag{23}$$

The above relation of octet-singlet mass matrix suggests the usual $\mathrm{SU}_V(3)$ prediction for the tangent of the mixing angle

$$\tan\Theta_T = \mp\sqrt{\frac{M_{f_2'} - \frac{4}{3}M_{K_2^*} + \frac{1}{3}M_{a_2}}{\frac{4}{3}M_{K_2^*} - \frac{1}{3}M_{a_2} - M_{f_2}}} = \mp 0.556 \approx \mp\frac{1}{\sqrt{3}}. \tag{24}$$

Here, we have identified $M_{f_2} = 1270$ MeV and $M_{f_2'} = 1525$ MeV respectively. This translates into a mixing angle $\Theta_T = 29.1^\circ$, which is close but not exactly the value $\theta_{\rm lin} = 26^\circ$ as quoted in Particle Data Group [11]. For comparison $\Theta_T = 35^\circ$ for an ideal mixing, and experimentally Θ_T is measured to be $(25.3 \pm 1.1)^\circ$ [12].

In the large N_c limit, quark loops are suppressed. Thus the leading diagrams in the meson sector contain a single quark loop. As a result, the octet and the singlet tensor mesons can be combined into a single *nonet* matrix

$$\tilde{\mathcal{N}}_{\mu\nu} = \Big(\tilde{\mathcal{O}} + \frac{\mathbf{1}}{\sqrt{3}}\tilde{\mathcal{S}}\Big)_{\mu\nu}. \tag{25}$$

The chiral Lagrangian in the large N_c limit is expressed exclusively in terms of $\tilde{\mathcal{N}}_{\mu\nu}$ tensor field. At leading order in $1/N_c$, the kinetic and interaction parts are given by

$$\begin{aligned} \mathcal{L}_{\rm kin} &= -\tfrac{i}{2}\,{\rm Tr}\tilde{\mathcal{N}}^\dagger_{\mu\nu}\, v\cdot\mathcal{D}\tilde{\mathcal{N}}^{\mu\nu} \\ \mathcal{L}_{\rm int} &= \frac{i\tilde{g}_2}{2}\,{\rm Tr}(\{\tilde{\mathcal{N}}^\dagger_{\mu\rho}, \tilde{\mathcal{N}}^\rho_\nu\}A_\lambda)v_\sigma\epsilon^{\mu\nu\lambda\sigma}, \end{aligned} \tag{26}$$

while the mass matrix part is given by

$$\mathcal{L}_{\rm mass} = \frac{\tilde{\mu}}{2}{\rm Tr}\tilde{\mathcal{N}}^\dagger_{\mu\nu}\tilde{\mathcal{N}}^{\mu\nu} + \frac{\tilde{\lambda}_2}{2}\,{\rm Tr}\Big(\{\tilde{\mathcal{N}}^\dagger_{\mu\nu}, \tilde{\mathcal{N}}^{\mu\nu}\}\mathcal{M}_\xi\Big). \tag{27}$$

Comparison with the chiral Lagrangian we have constructed above indicates that in the $N_c \to \infty$ limit

$$\Delta\tilde{\mu} \to 0, \quad \tilde{\sigma}_0 \to 0, \quad \tilde{\sigma}_8 \to 0, \tag{28}$$

$$\tilde{g}_1 \to \frac{2\tilde{g}_2}{\sqrt{3}}, \quad \tilde{\lambda}_1 \to \frac{2\lambda_2}{\sqrt{3}}, \quad \tan\Theta_T \to \frac{1}{\sqrt{2}}. \tag{29}$$

This means that $|f_2\rangle$ state becomes 'pure' $(s\bar{s})$ state and the nonet matrix is given by

$$\mathcal{N}_{\mu\nu} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2}{\sqrt{2}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2}{\sqrt{2}} & K_2^{*0} \\ K_2^{*-} & \overline{K}_2^{*0} & f_2' \end{pmatrix}. \tag{30}$$

In leading order one has the ideal mixing angle $(\Theta_T)_{N\to\infty} = 35^\circ$ which is close but off by 20% from the actual value $\Theta_T = 29.1^\circ$ obtained from Eq. (24). Note, however, that the mixing angle is quite sensitive to the f_2 masses, and Eq. (24) can be brought into consistency with ideal mixing by just shifting $f_2'(1525)$ up by 40 MeV. This discrepancy may be due to contamination of $f_2 - f_2'$ tensor mesons with exotic states such as tensor glueball states and/or exotic mesons such as $\rho - \rho$, $\omega - \omega$ bound-state resonances [13].

The coupling constants $\tilde{g}_{1,2}$ are free parameters in chiral perturbation theory. To estimate these coupling constants, we decompose the quark-model wave functions of vector and tensor mesons in their rest frames ($v = (1, \vec{0})$) into their isospin, spin and spatial parts.

$$\begin{aligned} |V^a(\epsilon^i)\rangle &= |\lambda^a\rangle \otimes |S=1, S_z=i\rangle \otimes |1s, L=0, L_z=0\rangle\,, \\ |T^a(\epsilon^{ij})\rangle &= |\lambda^a\rangle \otimes |S=1, S_z=i\rangle \otimes |2p, L=1, L_z=j\rangle + (i \leftrightarrow j) - \text{trace in } (ij) \end{aligned} \tag{31}$$

Note that the vector and tensor mesons are different only in their spatial wave functions. Current algebra [14, 15] suggests that the axial current coupling is given by

$$A^{ai} \sim V \lambda^a \mathbf{S}^i V^\dagger\,, \tag{32}$$

where

$$V = \exp(-i\theta(\mathbf{S} \times \mathbf{L})_z)\,, \tag{33}$$

with θ a small parameter ($\sin^2\theta \sim \frac{1}{8}$ in the baryon sector, and is expected to be of similar magnitude for mesons). In the zeroth order of an expansion in θ, the axial current coupling $A^{ai} = \lambda^a S^i$ does not couple to the spatial wave functions at all. As a result, $g_{1,2} = \tilde{g}_{1,2}$, *i.e.*, the same parameters govern the axial couplings of the vector and tensor mesons. As discussed in [7], under the assumption of ideal mixing, $\tilde{g}_1 = g_1 = 2/\sqrt{3}$ and $\tilde{g}_2 = g_2 = 1$ in the nonrelativistic constituent quark model. If we employ the non-relativistic chiral quark model [16], these factors are further reduced by a factor of 3/4.

In chiral perturbation theory, the leading order corrections to the tensor meson masses are of order $m_q^{3/2}$ and arise from one-loop self-energy diagrams due to virtual pseudoscalar meson exchange. A straightforward calculation yields

$$\begin{aligned} \delta M_{a_2} &= -\frac{1}{8\pi f^2}\Big[\tilde{g}_2^2\Big(\frac{2}{3}m_\pi^3 + 2m_K^3 + \frac{2}{3}m_\eta^3\Big) + \tilde{g}_1^2 m_\pi^3\Big] \\ \delta M_{K_2^*} &= -\frac{1}{8\pi f^2}\Big[\tilde{g}_2^2\Big(\frac{3}{2}m_\pi^3 + \frac{5}{3}m_K^3 + \frac{1}{6}m_\eta^3\Big) + \tilde{g}_1^2 m_K^3\Big] \\ \delta M_{11}^{(08)} &= -\frac{1}{8\pi f^2}\tilde{g}_1^2\Big(3m_\pi^3 + 4m_K^3 + m_\eta^3\Big) \\ \delta M_{22}^{(08)} &= -\frac{1}{8\pi f^2}\Big[\tilde{g}_2^2\Big(2m_\pi^3 + \frac{2}{3}m_K^3 + \frac{2}{3}m_\eta^3\Big) + \tilde{g}_1^2 m_\eta^3\Big] \\ \delta M_{12}^{(08)} &= \delta M_{21}^{(08)} = +\frac{1}{8\pi f^2}\sqrt{\frac{2}{3}}\tilde{g}_1\tilde{g}_2(-3m_\pi^3 + 2m_K^3 + m_\eta^3)\,. \end{aligned} \tag{34}$$

The mass corrections, which are of $\mathcal{O}(N_c^{-1})$, are quite substantial numerically. For example, $\delta m_{a_2} \approx -450$ MeV.

The large mass corrections Eq. (34), which are of $\mathcal{O}(p^3)$ in chiral perturbation expansion, are essentially of the same structures as for vector mesons studied in [7]. For vector mesons, such large mass corrections are problematic since it breaks experimentally observed equal spacing relations such as $m_\phi - m_{K^*} = m_{K^*} - m_\rho$ within 10 % error. This problem has been addressed in [17], where it was pointed out that higher-order chiral corrections at $\mathcal{O}(p^4)$ repairs the problem. For tensor mesons, one again finds a phenomenological mass relation $m_{K_2^*} - m_{f_2} = m_{K_2^*} - m_{a_2}$ within 12 % error. The mass corrections Eq. (34) again break this phenomenological relation sizably. Nevertheless, since $\mathcal{O}(p^4)$ corrections as well as the leading nonanalytic corrections Eq. (34) at $\mathcal{O}(p^3)$ are essentially of the same structures as for vector mesons, we expect the problem would be repaired likewise. Moreover, we have ignored finite decay width of tensor mesons so far. At large N_c, the decay widths become narrow and may be ignored. At $N_c = 3$, the observed decay widths are quite sizable: $\Gamma_{f_2} \sim 180 \pm 20$ MeV, $\Gamma_{a_2} \sim 110$ MeV, and $\Gamma_{K_2^*} \sim 100$ MeV. Effects of the finite decay width can be systematically incorporated as non-Hermitian terms in Eq. (13) in chiral perturbation theory. Interestingly, decay widths of tensor mesons are numerically uniform and relatively smaller than those for vector mesons. Indeed, inferring from previous works on such effects for vector mesons [18, 19], rough numerical estimate indicates that the effect would not ruin validity of chiral perturbation theory.

In contrast to mass corrections, the singlet-octet mixing angle Θ_T after leading order chiral perturbation one-loop correction is taken into account remains very small. The mixing angle is

$$\tan\Theta_T = \mp\sqrt{\frac{M_{f_2'} - \frac{4}{3}M_{K_2^*} + \frac{1}{3}M_{a_2} - \delta M}{\frac{4}{3}M_{K_2^*} - \frac{1}{3}M_{a_2} - M_{f_2} + \delta M}}\,, \tag{35}$$

where

$$\begin{aligned}\delta M &= -\frac{4}{3}\delta M_{K_2^*} + \frac{1}{3}\delta M_{a_2} + \delta M_{22}^{(08)} \\ &= -\frac{1}{8\pi f^2}\Big(\tilde{g}_1^2 + \frac{2}{3}\tilde{g}_2^2\Big)\Big(\frac{1}{3}m_\pi^3 - \frac{4}{3}m_K^3 + m_\eta^3\Big)\,.\end{aligned} \tag{36}$$

Using the large N_c relation between $\tilde{g}_1$ and $\tilde{g}_2$, we find that

$$\delta M \approx -\frac{2\tilde{g}_2^2}{8\pi f^2}\Big(\frac{1}{3}m_\pi^3 - \frac{4}{3}m_K^3 + m_\eta^3\Big)\,. \tag{37}$$

For a reasonable range $\tilde{g}_2 \approx 0.75 - 1$, the mass correction $\delta M \approx -6$ MeV. The corresponding shift in mixing angle is about 1°. Its smallness when contrasted with the huge corrections to individual masses) is because the combination δM transforms as **27** under flavor $SU(3)$, hence, should be numerically small, much as in chiral corrections to Gell-Mann-Okubo relation of baryon masses.

A remark is in order. Various heuristic arguments indicate that $J = 2$ tensor mesons might exhibit moderate mixing with other 2^{++} meson states. The first is tensor glueball. In the past, there has been various phenomenological model for the mixing. It has been argued that, once the glueball mixing is taken into account, the f_2 meson is in ideal mixing [12]. The second are tetra-quark states. There has been no systematic analysis of their influence to the mixing. Within chiral perturbation theory the tetraquark states might be treatable systematically as a perturbation of $(\mathbf{8}_L, \mathbf{8}_R) \subset (\mathbf{8}_L, \mathbf{8}_R) \otimes (\mathbf{8}_L, \mathbf{8}_R)$ irreducible state. Electromagnetic correction to the mass matrix is of theoretical interest. In this case one has to bear in mind that both short-distance and long-distance effects contributions have to be taken into account.

It is straightforward to generalize the chiral perturbation theory to $(q\overline{q})$ mesons of higher spin. The spin-s meson fields are introduced in terms of 3×3 octet matrix $\mathcal{O}_{\mu_1\mu_2\cdots\mu_s}$ and a singlet $\mathcal{S}_{\mu_1\mu_2\cdots\mu_s}$. The spin-$s$ mesons are irreducible representations of the Lorentz group, viz. totally symmetric and traceless components. The polarization of these mesons are necessarily orthogonal to the momentum. For heavy static fields the four-velocity also satisfies $v^{\mu_1}\mathcal{O}_{\mu_1\mu_2\cdots\mu_s} = v^{\mu_1}\mathcal{S}_{\mu_1\cdots\mu_s} = 0$. Chiral Lagrangian of heavy spin-s tensor mesons is exactly the same as Eqs.(13)-(18):

$$
\begin{aligned}
\mathcal{L}_{\rm kin} &= -\frac{i}{s!}\mathcal{S}^\dagger_{\mu_1\cdots\mu_s}(v\cdot\partial)\mathcal{S}^{\mu_1\cdots\mu_s} - \frac{i}{s!}\mathrm{Tr}\,\mathcal{O}^\dagger_{\mu_1\cdots\mu_s}(v\cdot\mathcal{D})\mathcal{O}^{\mu_1\cdots\mu_s} \\
\mathcal{L}_{\rm int} &= \begin{cases} 0, & s=0; \\ \frac{i}{s!}g_1\mathcal{S}^\dagger_{\mu\rho_1\cdots}\mathrm{Tr}\,(\mathcal{O}^{\rho_1\cdots}_\nu A_\lambda)v_\sigma\epsilon^{\mu\nu\lambda\sigma} + (h.c.) & \\ \qquad +\frac{i}{s!}g_2\mathrm{Tr}(\{\mathcal{O}^\dagger_{\mu\rho_1\cdots},\mathcal{O}^{\rho_1\cdots}_\nu\}A_\lambda)v_\sigma\epsilon^{\mu\nu\rho\sigma}, & \text{otherwise,} \end{cases} \\
\mathcal{L}_{\rm mass} &= \frac{\overline{\mu}_0}{s!}\mathcal{S}^\dagger_{\mu_1\cdots\mu_s}\mathcal{S}^{\mu_1\cdots\mu_s} + \frac{\overline{\mu}_8}{s!}\mathrm{Tr}\mathcal{O}^\dagger_{\mu_1\cdots\mu_s}\mathcal{O}^{\mu_1\cdots\mu_s} \\
&\quad +\frac{\lambda_1}{s!}\mathrm{Tr}(\mathcal{O}^\dagger_{\mu_1\cdots\mu_s}\mathcal{M}_\xi)\mathcal{S}^{\mu_1\cdots\mu_s} + (h.c.) + \frac{\lambda_2}{s!}\mathrm{Tr}(\{\mathcal{O}^\dagger_{\mu_1\cdots\mu_s},\mathcal{O}^{\mu_1\cdots\mu_s}\}\mathcal{M}_\xi)\,. \qquad (38)
\end{aligned}
$$

Since the interaction Lagrangian structure is exactly the same as 2^{++} tensor mesons, the leading order and the one-loop correction to mass spectra and mixing angles follow exactly the same pattern. Hence the singlet-octet mixing for these higher spin mesons are also related to their masses through Eq. (24), and be ideally mixed in the large N_c limit. For example, by plugging in the masses of the spin-3 mesons one obtains the mixing angle $\Theta = 28°$, in qualitative agreement with the large N_c prediction. Moreover, the chiral one-loop correction will take the form of Eq. (35) where δM will be given by Eq. (37) up of spin-dependent multiplicative constants. Note that, however, these higher spin mesons are heavier in mass and there are many other possible sources of contamination, which cannot be treated within the framework of chiral perturbation theory.

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Throughout the discussion, we have taken the chiral coupling g_2 to a reasonable value motivated by large N_c limit and non-relativistic quark model arguments. The corresponding g_2 for vector meson chiral perturbation theory has been determined to be $g_2 \simeq 0.57$ in [19] from experimentally observed $\tau \to \rho\pi\nu_\tau$ or $\tau \to K^*\pi\nu_\tau$ decays. Within large N_c limit and non-relativistic quark model, the g_2 coupling constant for all higher spin mesons is expected to be of similar value. Unfortunately, corresponding decay modes are not measured experimentally yet and therefore direct determination of the coupling is currently out of reach.

In this paper, we have studied chiral perturbation theory of heavy tensor mesons of spin ≥ 2. We have shown that the octet - singlet mixing angle is quite close to 'ideal mixing' and one-loop correction to the mixing angle is negligible. On the other hand, mass spectra themselves receive quite a sizable corrections. We conclude that the chiral perturbation theory 'explains' naturally why the singlet-octet mixing is signficantly off the ideal mixing for pseudo-scalar Goldstone bosons while close to the ideal mixing for all higher spin ≥ 1 mesons. As a final remark, it would be of interest to extend the present analysis to electromagnetic contributions to tensor masses and mixing angles. For vector mesons, such an analysis has been undertaken recently [20] and have found several new sources of $\rho - \omega$ mixing coming from short-distance effects. Our formalism suggests that their results go through directly to tensor messons as well. It would be very interesting to check this explicitly.

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